

PC Notes Unit 6 Section 5.1 & 5.2 Using Fundamental Identities & Verifying Trig Identities

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Identities

Odd

$$\sin(-x) = -\sin(x) \quad \csc(-x) = -\csc(x)$$

$$\tan(-x) = -\tan(x) \quad \cot(-x) = -\cot(x)$$

Even

$$\cos(-x) = \cos(x)$$

$$\sec(-x) = \sec(x)$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

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Identities can be used to find other trig values.

- 1) Find the identity that relates the function given with the one needed.
- 2) Plug in the value and solve for the unknown value.
- 3) Use other identities to derive the rest of the values.

Example: Given $\sec x = \frac{-3}{2}$ and $\tan x > 0$, use an identity to find the other five trig values of x .

1. $\tan^2 \theta + 1 = \sec^2 \theta$ **or** $\tan^2 \theta = \sec^2 \theta - 1$

$$\tan^2 \theta = \frac{3^2}{2^2} - 1$$

2. $\tan^2 \theta = \frac{9}{4} - \frac{4}{4}$

$$\tan \theta = \pm \frac{\sqrt{5}}{2} \text{ since } \tan \theta > 0 \text{ then } \tan \theta = \frac{\sqrt{5}}{2}$$

3. **Reciprocal identities:** If $\sec \theta = -\frac{3}{2}$ then $\cos \theta = -\frac{2}{3}$. If $\tan \theta = \frac{\sqrt{5}}{2}$ then $\cot \theta = \frac{2}{\sqrt{5}}$.

Pythagorean Identities: use

$$\begin{array}{l} \text{R} \\ \cos^2 \theta + \sin^2 \theta = 1 \\ \sin^2 \theta = 1 - \frac{4}{9} \\ \text{S} \\ \sin^2 \theta = \frac{9}{9} - \frac{4}{9} \\ \sin^2 \theta = \frac{5}{9}, \text{ so } \sin \theta = \pm \frac{\sqrt{5}}{3} \\ \text{T} \\ \sin \theta = -\frac{\sqrt{5}}{3}, \text{ csc } \theta = -\frac{3}{\sqrt{5}} \end{array}$$

How do you decide whether $\sin \theta$ is + or -? Determine the Quadrant based on info given $\sec \theta < 0, \tan \theta > 0$. Must be QIII so $\sin \theta < 0$

Other option – Use info given to draw rt triangle with reference angle θ , find x,y,r

To simplify trig expressions means to write it as a number or in terms of a single trig function, if possible.

Simplify the following:

1) $\frac{\tan^2 \theta}{\sec^2 \theta}$

2) $\frac{\tan^2 \theta}{\sec \theta - 1}$

$$3) \frac{\tan \theta(1 + \cot^2 \theta)}{\cot \theta}$$

$$4) \sin \theta (\cos \theta + \sin \theta \tan \theta)$$

$$5) \frac{\sec x - \cos x}{\sin^2 x \sec^2 x}$$

Review: Perform each indicated operation and simplify the results

$$1) (x + 4)^2$$

$$2) (3x - 5)^2$$

$$3) (\cos x + 1)^2$$

$$4) (\sin x - \cos x)^2$$

$$5) \sin^2 \theta (\cot^2 \theta + 1)$$