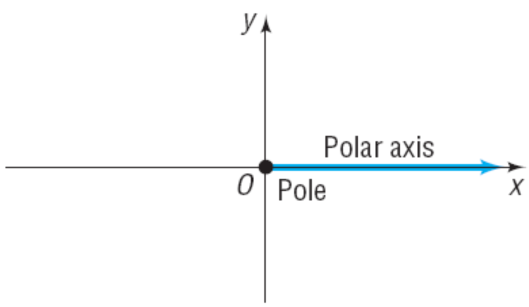
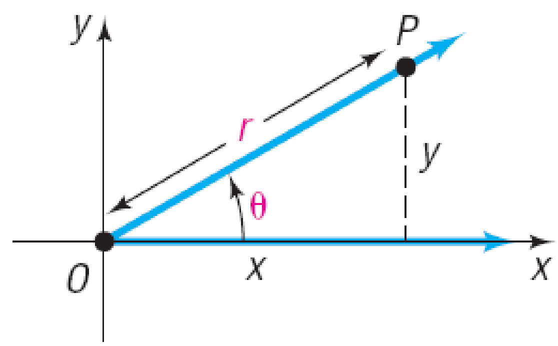
PC Notes Unit 6 Section 9.6 and 6.5 Polar Form

Objectives: **Plot points using polar coordinates; convert from rectangular to polar and polar to rectangular; transform equations from polar to rectangular form; write complex numbers in polar form (cis)**

In this section we will plot points and graph equations in the polar coordinate system.

Points are plotted as an (r,θ) pair where r is the distance from the pole (origin) and θ is the location of the terminal side of an angle in standard position.





Because coterminal angles share the same terminal side, a point given in polar form is not unique. There is more than one way to express a point as an ordered pair in polar form.

To plot polar coordinates:

- **Find the terminal side of angle θ, then find the point that is a distance of r units from the origin along that angle.**
- **If r is negative, plot the point that is r units on the opposite ray from angle θ.**

Converting from Polar (r,θ) to Rectangular form (x,y)

x=rcosθ y=rsinθ

To convert from rectangular to polar: $r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Transform Equations from Polar to Rectangular Form

Common techniques for transforming an equation from polar to rectangular form are:

- **Multiply both sides by r**
- **Square both sides of the equation**
- **Look for possible substitutions such as**
  $x^2 + y^2 = r^2$, $x = r\cos\theta$ and $y = r\sin\theta$

Complex Numbers: a+bi can be represented graphically as a point (a,b) or as an arrow from the origin to the point, where a (the real part) is plotted on the horizontal axis and b (the imaginary part) is plotted on the vertical axis.

The length of the arrow is $\sqrt{a^2+b^2}$ **, which is called the absolute value of the complex number.**
To convert a complex number to polar form: z=a+bi

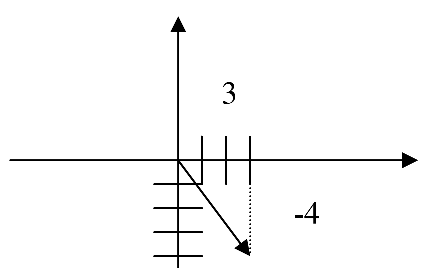
z=rcosθ +(rsinθ)i =r(cosθ+i sinθ) where r is the magnitude of z and θ is the argument of z (either in degrees or radians)

$$r = \sqrt{a^2+b^2} \quad \text{and } \theta = \tan^{-1}\left(\frac{b}{a}\right)$$

For convenience an abbreviation is used as in the following example:

Transform z=3 – 4i into polar form.

Sketch graph



$$r = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{-4}{3}\right) = 307°$$

$$z = 5\cos 307° + i\sin 307° = 5cis307°$$

Properties of Complex Numbers in Polar Form

Product of two complex numbers: $z_1 z_2 = r_1 r_2 cis(\theta_1 + \theta_2)$

Reciprocal of a complex number: $\frac{1}{z} = \frac{1}{r} cis(-\theta)$

Quotient of two complex numbers: $\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$