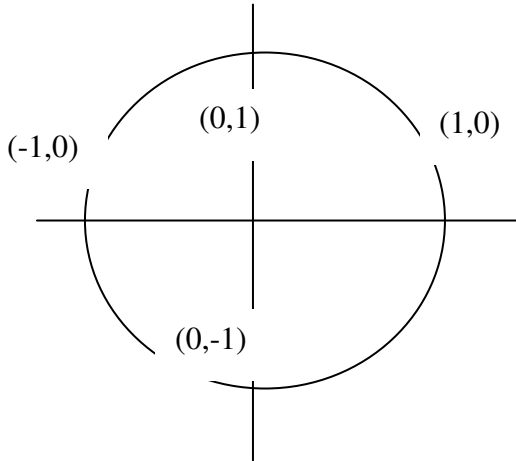


Notes on 4-2 The Unit Circle

Unit Circle = circle with center at the origin and a radius of 1.



Its equation is $x^2 + y^2 = 1$

$$x = \cos \theta \quad y = \sin \theta$$

which is where the Pythagorean Identity comes from $\cos^2 x + \sin^2 x = 1$

The coordinates of the ordered pair correspond to both an angle θ and an arc length t .

In circular trigonometry $\tan \theta = \frac{y}{x}$. The rest of the trig functions are the

reciprocals: $\sec \theta = \frac{1}{x}$, $\csc \theta = \frac{1}{y}$, $\cot \theta = \frac{x}{y}$.

Periodic Properties

A function f is called **periodic** if there is a positive number p such that $f(\theta + p) = f(\theta)$. If there is a smallest such number p , this smallest value is the **period** of the function f .

The six trig functions are all periodic – they repeat their values over and over in a regular pattern.

- Sine, cosine, cosecant and secant have period 2π
- Tangent and cotangent have period π

$$\begin{array}{lll} \sin(\theta + 2\pi) = \sin \theta & \cos(\theta + 2\pi) = \cos \theta & \tan(\theta + \pi) = \tan \theta \\ \csc(\theta + 2\pi) = \csc \theta & \sec(\theta + 2\pi) = \sec \theta & \cot(\theta + \pi) = \cot \theta \end{array}$$

Even-Odd Functions

- Cosine and secant are even functions (symmetric with respect to y-axis)
- All the others are odd functions (symmetric with respect to the origin)

Odd $f(-x)=-f(x)$	Even $f(-x)=f(x)$	Odd
$\sin(-\theta) = -\sin \theta$	$\cos(-\theta) = \cos \theta$	$\tan(-\theta) = -\tan \theta$
$\csc(-\theta) = -\csc \theta$	$\sec(-\theta) = \sec \theta$	$\cot(-\theta) = -\cot \theta$

Domain and Range

Since we now can define these terms as a function of t , where t can be any real number (representing the arc length starting at $(1,0)$ and wrapping around the Unit Circle), we will discuss Domain and Range for each trig function.

Remember **Domain** is restricted by whatever angle or arc length would cause the function to be undefined (division by 0). **Range** is the actual trig value.

Function	Symbol	Domain	Range
sine	$f(\theta) = \sin \theta$	All real numbers	All real numbers from -1 to 1 , inclusive
cosine	$f(\theta) = \cos \theta$	All real numbers	All real numbers from -1 to 1 , inclusive
tangent	$f(\theta) = \tan \theta$	All real numbers, except odd multiples of $\frac{\pi}{2}$ (90°)	All real numbers
cosecant	$f(\theta) = \csc \theta$	All real numbers, except integer multiples of π (180°)	All real numbers greater than or equal to 1 or less than or equal to -1
secant	$f(\theta) = \sec \theta$	All real numbers, except odd multiples of $\frac{\pi}{2}$ (90°)	All real numbers greater than or equal to 1 or less than or equal to -1
cotangent	$f(\theta) = \cot \theta$	All real numbers, except integer multiples of π (180°)	All real numbers