Section 10.1 The Language of Hypothesis Testing

Objective: Identify the null and alternate hypotheses in a statistical test and recognize the types of errors.

Statisticians draw inferences about population parameters based on samples. Two types of inferences:

- **Estimate the value of a parameter (Chapter 9)**
- **Hypothesis testing (Chapter 10)**

We want to test whether a particular claim is believable or not—the process that we use is called hypothesis testing. Hypothesis testing involves two steps

- **Step 1 – state what we think is true**
- **Step 2 – quantify how confident we are in our claim (level of significance)**

Hypothesis Testing:

- **We state our claim with the null hypothesis - H_0 - the statement to be tested (read as "H – naught")**
- **Our counter-claim is called the alternative hypothesis - H_1 - is the opposite of the statement to be tested (read as "H-one")**

There are three ways to set up null and alternative hypotheses.

1. **A two-tailed test, tests whether the parameter is either equal to versus not equal to some value.**
   - H_0: **parameter = some value**
   - H_1: **parameter ≠ some value**

Example: Two-Tailed Test

A company manufactures precision ball bearings, and the average diameter should be 6.0 mm. We suspect the average of the ball bearings is not what is advertised. We do not know if it is less than or greater than, we just suspect it is different.

$$H_0 : Diameter = 6\ mm$$

$$H_1 : Diameter \neq 6\ mm$$

An alternative hypothesis of "≠ 6" is appropriate since a sample diameter that is too high is a problem and a sample diameter that is too low is also a problem. Thus, this is a two-tailed test.

2. **A left-tailed test, tests whether the parameter is either equal to, versus less than, some value.**
   - H_0: **parameter = some value**
   - H_1: **parameter < some value**

Example: Left-Tailed Test

A car manufacturer claims that the mileage for a certain make of car is at least 29 mpg. We suspect that the mileage is actually less than that and that they are over rating the mileage.

$$H_0 : MPG = 29$$

$$H_1 : MPG < 29$$

An alternative hypothesis of " < 29" is appropriate since a mpg that is too low is a problem but a mpg that is too high is not a problem. Thus, this is a left-tailed test.

3. **A right-tailed test, tests whether the parameter is either equal to, versus greater than, some value.**
   - H_0: **parameter = some value**
   - H_1: **parameter > some value**

Example: Right-Tailed Test

A bolt manufacturer claims that the defective rate of their product is at most 1 part in 1,000. We suspect that the defect rate is actually higher than stated by the manufacturer.

$$H_0 : Defect\ Rate = 0.001$$

$$H_1 : Defect\ Rate > 0.001$$

An alternative hypothesis of " > 0.001" is appropriate since a defective rate that is too low is not a problem, but a defect rate that is too high is a problem. Thus, this is a right-tailed test.

Work #1

There are two possible results for a hypothesis test

- **If we believe that the null hypothesis could be true, this is called not rejecting the null hypothesis (note: we are not accepting the null hypothesis as true, there just isn't enough evidence to reject)**
- **If we are pretty sure that the null hypothesis if not true, so that the alternative hypothesis is true, this is called rejecting the null hypothesis**

Summary of Errors in Hypothesis Testing

		Reality	
		H_0 Is True	H_1 Is True
Conclusion	Do Not Reject H_0	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

We see that there are four possibilities … in two of which we are correct and in two of which we are incorrect.

- **When we reject (and state that the null hypothesis is false) but the null hypothesis is actually true, this is called a Type I error.**
  - **The level of significance, α, is the probability of making a Type I error.**
  - **α is chosen by the researcher before the sample data are collected.**
  - **If the consequences are severe, the level of significance should be small (say $\alpha = 0.01$)**
- **When we do not reject (and state the null hypothesis could be true) but the null hypothesis is actually false, this is called a Type II error.**
  - **β is the probability of making a Type II error.**
- **In general, Type I errors are considered the more serious of the two.**

Work #2 - 4