Section 9.1 The Logic of Constructing Confidence Intervals about a Population Mean

Objective: Determine a confidence interval for a mean; determine the minimum sample size necessary for determining a confidence interval for a mean.

A *point estimate* is the value of a statistic that estimates the value of a parameter.

- **$\bar{x}$ is used as a point estimate for μ.**
- **s is used as a point estimate for σ.**

A *confidence level*, *c*, is a measure of the degree of assurance we have in our point estimate.

- **Theoretically, the value of *c* may be any number between zero and one.**
- **Typical values for *c* include 0.90, 0.95, and 0.99.**
- **$z_{\frac{\alpha}{2}}$ is called the critical value for a confidence level *c*. (Remember, z_{α} is the *z*-score such that the area under the standard normal curve to the <u>right</u> of z_{α} is α, so use the positive z-score.)**
- **Confidence interval estimates for the population mean are of the form *point estimate* ± *margin of error***

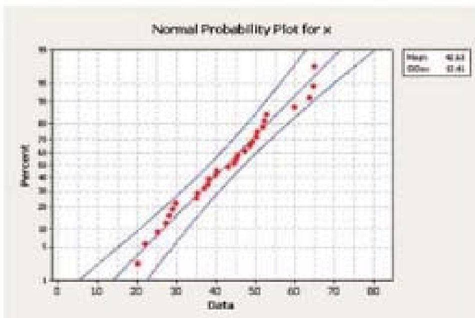
The *margin of error*, *E*, depends on :

- **Level of confidence – as the level of confidence increases, the margin of error also increases.**
- **Sample size – as the sample size increases, the margin of error decreases (Law of Large Numbers)**
- **Standard deviation – an increase in the standard deviation will widen the confidence interval.**

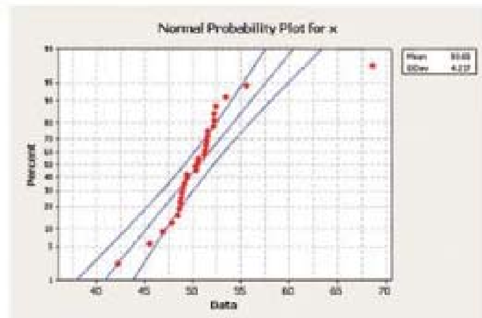
Work #1 – 2

Requirements for Constructing a Confidence Interval (Z-Interval):

- **The data is from a random sample.**
- **The population standard deviation, σ, is assumed to be known.**
- **Sample size must be large ($n \geq 30$) or the population must be normally distributed.**
  - **If small sample size, all data values must lie within the bounds on the normal probability plot. In other words, it must roughly linear.**
  - **A boxplot of the data values does not show any outliers.**



Normally Distributed



Not Normally Distributed

Confidence Interval for μ (large samples, $n \geq 30$)

$$\bar{x} - E < \mu < \bar{x} + E$$

where $\bar{x}$ **= sample mean**

$$E = z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$ **is the Margin of Error**

c **= confidence level** $(0 < c < 1)$

$z_{\frac{\alpha}{2}}$ **= critical value (from table in appendix)**

n **= sample size** $(n \geq 30)$

We have a more precise (smaller) interval if

- **Increase sample size (the difference between the population mean and the sample mean decreases)**
- **Decrease the level of confidence**

Interpretation of a Confidence Interval –
"We are _____% confident that the interval actually does contain the true value of the population mean."

Using TI 83/84 to Construct Confidence Intervals

If Given Summary Statistics (μ and σ):

- **Press STAT, highlight TESTS, and select 7:ZInterval**
- **Highlight Stats**
- **Enter σ, $\bar{x}$, n, C-Level**
- **Press Enter on Calculate**

The procedures we are using for constructing confidence intervals are robust, which means minor departures from normality will not seriously affect the results. As sample size increases the mean is more resistant to outliers, thus the confidence interval will be more robust.

Work #3 – 5

Determining the Sample Size *n*

The sample size required to estimate the population mean, μ, with a specified margin of error, *E*, is given by

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E} \right)^2$$

where *n* is round up to the nearest whole number.

Work #6