Section 7.4 The Fundamental Theorem of Calculus

An indefinite integral is a family of functions in which all the functions are antiderivatives of a function f. A definite integral is a real number. The definite integral $\int_a^b f(x)\,dx$ gives the total change of $F(x)$ as x changes from a to b.

FUNDAMENTAL THEOREM OF CALCULUS

Let f be continuous on the interval $[a, b]$, and let F be *any* antiderivative of f. Then

$$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b.$$

PROPERTIES OF DEFINITE INTEGRALS

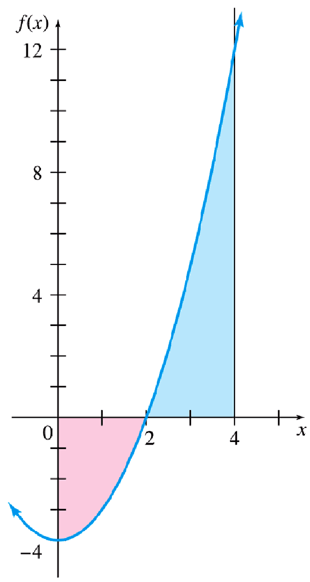
If all indicated definite integrals exist,

1. $\int_a^a f(x)\,dx = 0$;
2. $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$ for any real constant k (constant multiple of a function);
3. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$ (sum or difference of functions);
4. $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for any real number c;
5. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$.

Work #1 - 3

Area

If $f(x) \geq 0$ in $[a, b]$, the definite integral $\int_a^b f(x)\,dx$ gives the area below the graph of the function $y = f(x)$ above the x-axis and between the lines $x = a$ and $x = b$. In the figure below, the definite integral over the interval below the x-axis will have a negative value. Therefore, to find the area, integrate the negative and positive portions separately and take the absolute value before combining the results to get the total area.



FINDING AREA

In summary, to find the area bounded by $f(x)$, $x = a$, $x = b$, and the x-axis, use the following steps.

1. Sketch a graph.
2. Find any x-intercepts of $f(x)$ in $[a, b]$. These divide the total region into subregions.
3. The definite integral will be *positive* for subregions above the x-axis and *negative* for subregions below the x-axis. Use separate integrals to find the (positive) areas of the subregions.
4. The total area is the sum of the areas of all of the subregions.

Work #4 - 6