Section 2.2 – 2.6 Review of Nonlinear Functions

A quadratic function in standard form is of the form $f(x) = ax^2 + bx + c$ where a, b, and c are real numbers and $a \neq 0$.

A quadratic function in vertex form is of the form $f(x) = a(x-h)^2 + k$ where (h, k) is the vertex.

- **The domain of a quadratic function is the set of all real numbers.**
- **Vertex is (h, k) where $h = -\frac{b}{2a}$ and $k = f(h)$**
- **Axis of symmetry is $x = h$**
- **The maximum or minimum occurs at the vertex and is the value of k**
  - **If $a > 0$, opens up and the vertex is a minimum point**
  - **If $a < 0$, opens down and the vertex is a maximum point**
- **x- and y-intercepts**
  - **x-intercepts: set $y = 0$, then solve for x by factoring or quadratic formula**
  - **y-intercept: set $x = 0$, solve for y**

Transformations can be applied to any function.

$$y = a \cdot f(x-h) + k \quad \text{where}$$

- **a (vertical stretch or compression) – multiply y by a**
  $a > 1$ graph stretched by a factor of a
  $0 < a < 1$ graph compressed by a factor of a

- **$-a$ (reflection about the x-axis) – multiply y by -1**
  If a is negative, the graph will be reflected over the x-axis.

- **h (horizontal shift) – move opposite the direction of the sign**
  $(x - h)$ graph shifts h units right
  $(x + h)$ graph shifts h units left

- **k (vertical shift) – move same direction as the sign**
  $+k$ graph shifts up k units
  $-k$ graph shifts down k units

$y = f(ax - h) + k$ where

- a **(horizontal stretch or compression) – divide x by a**
  $0 < a < 1$ graph stretched horizontally by a factor of a
  $a > 1$ graph compressed horizontally by a factor of a
  $-a$ (reflection about the y-axis) – multiply x by -1

Work #1 – 5

A polynomial function is a sum of terms Ax^n where A are real numbers and n is a nonnegative integer.

- **The domain is the set of all real numbers.**
- **The degree is the largest power of x.**

Graphing Polynomials

We will use the end behavior of the power function to determine what happens to a polynomial graph as x becomes very large or very small.

- **If the degree of the polynomial is odd, the ends will point in opposite directions.**
  - **Positive leading coefficient: down on left / up on right**
  - **Negative leading coefficient: up on left / down on right**

- **If the degree of the polynomial is even, the ends will point in the same direction.**
  - **Positive leading coefficient: up on left / up on right**
  - **Negative leading coefficient: down left / down right**

A rational function is a function of the form $R(x)=\dfrac{p(x)}{q(x)}$ where p and q are polynomials and $q(x)\neq 0$.

To analyze the graph of a rational function, find the following:

- **Find the domain – set the denominator equal to zero and solve. The domain is all real numbers except for the solutions you have just found.**
- **Vertical asymptotes – the solutions you just found for the domain give you the vertical asymptotes: $x = a_1, x = a_2$**
- **Horizontal asymptotes – remember, these are based on the relationship between the degree of the numerator and the degree of the denominator.**
    - **If degree of numerator < degree denominator, $y = 0$.**
    - **If degree of numerator = degree denominator, $y=\dfrac{a}{b}$**
    - **If degree of numerator > degree denominator by one, the graph has an oblique (slant) asymptote.**
- ***x*-intercepts: set numerator = 0, then solve**
- ***y*-intercept: replace all *x*'s with 0, solve for *y***
- **Use a sign chart to graph the function.**
    - **Plot vertical asymptotes and *x*-intercepts on a number line.**
    - **Choose a test point in each interval on the number line and determine if test point solution is positive or negative.**
    - **If positive – graph is above the x-axis in the interval.
If negative – graph below the x-axis in the interval.**
- **Check for behavior near intercepts (multiplicity).**
- **In a rational function, if the numerator and the denominator have a common factor, the rational expression can be reduced and the graph will have a "hole" at the x-value of the common factor.**
    - **The domain of the rational function is determined <u>before</u> the function is simplified.**
    - **Other analysis of graph is done after the function is simplified.**
- **If the degree of the numerator is one greater than the degree of the denominator, then the rational function has an oblique asymptote.**
    - **Use division to rewrite the function as $quotient + remainder / divisor$.
The equation $y = quotient$ is an oblique asymptote.**

Work #6 – 8

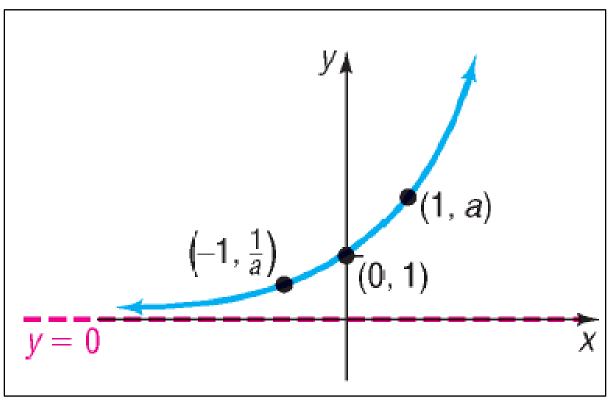
Review of Exponentials

Exponential functions may be the single most important type of functions used in practical applications. They are used to describe growth and decay, which are important ideas in management, social science, and biology.

An exponential function is a function of the form $f(x) = a^x$ where $a > 0$ and $a \neq 1$.

Properties of exponential functions, $f(x) = a^x$, for $a > 1$

- **Domain is the set of all real numbers; the range is the set of positive real numbers.**
- **There are no x-intercepts; the y-intercept is 1.**
- **The x-axis ($y = 0$) is a horizontal asymptote.**
- **$f(x) = a^x$ where $a > 0$ is an increasing function and one-to-one.**
- **The graph is smooth and continuous.**
- **The graph contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.**



DEFINITION OF *e*

As m becomes larger and larger, $\left(1 + \frac{1}{m}\right)^m$ becomes closer and closer to the number e, whose approximate value is 2.718281828.

COMPOUND AMOUNT

If P dollars is invested at a yearly rate of interest r per year, compounded m times per year for t years, the **compound amount** is

$$A = P\left(1 + \frac{r}{m}\right)^{tm} \text{ dollars.}$$

CONTINUOUS COMPOUNDING

If a deposit of P dollars is invested at a rate of interest r compounded continuously for t years, the compound amount is

$$A = Pe^{rt} \text{ dollars.}$$

Solving Exponential Equations

When two exponential expressions with the same base are equal, then their exponents are equal.

Work #9 - 14

Review of Logarithms

The inverse of the exponential function is called the logarithmic function.

LOGARITHM

For $a > 0$, $a \neq 1$, and $x > 0$,

$$y = \log_a x \qquad \text{means} \qquad a^y = x.$$

In general, $\log_a x$ means "what power must you raise a to in order to obtain x." In other words, a logarithm is an exponent.

The Natural Logarithm Function

$y = \log_e x$ **is called the natural logarithm function and is written** $y = \ln x$**, where ln is the abbreviation for the natural log. It is the inverse of the natural exponential function.**

Domain of Logarithmic Functions

The domain of a logarithmic function consists of positive real numbers, therefore, the argument of a log function must be greater than zero. In other words, you cannot take the logarithm of zero or a negative number.

PROPERTIES OF LOGARITHMS

Let x and y be any positive real numbers and r be any real number. Let a be a positive real number, $a \neq 1$. Then

a. $\log_a xy = \log_a x + \log_a y$

b. $\log_a \frac{x}{y} = \log_a x - \log_a y$

c. $\log_a x^r = r \log_a x$

d. $\log_a a = 1$

e. $\log_a 1 = 0$

f. $\log_a a^r = r$.

Change of Base Formula

- **Change-of-Base Formula (to base 10) :** $\log_b M = \frac{\log M}{\log b}$
  - $\log_5 89 = \frac{\log 89}{\log 5} = 2.789$
- **Change-of-Base Formula (to base *e*):** $\log_b M = \frac{\ln M}{\ln b}$
  - $\log_5 89 = \frac{\ln 89}{\ln 5} = 2.789$

Work #15 - 25