Section 10.4 Vectors

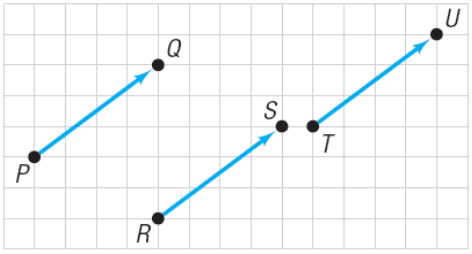
Objectives: Graph vectors; find a position vector; add and subtract vectors; find a unit vector; find a vector from its direction and magnitude.

A **vector** is a directed line segment—it has both magnitude and direction. The length of the segment represents the magnitude and the arrowhead represents the direction.

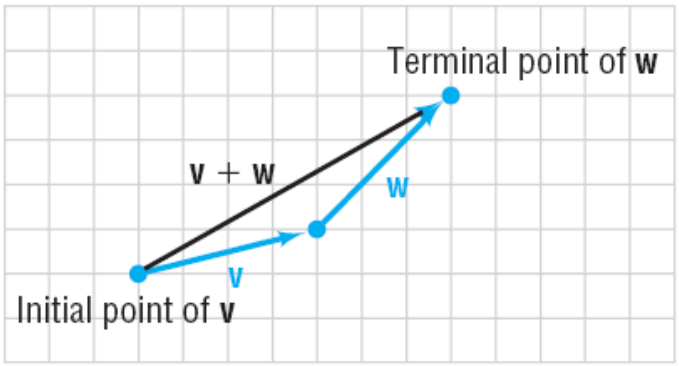
There are several ways to name a vector:

- Boldface type, $\mathbf{PQ}$
- Lowercase letter, v
- Two uppercase letters, $\overrightarrow{PQ}$, where P is the initial point and Q is the terminal point.

Two vectors are equal if they have the same magnitude and the same direction. **Therefore, the location of a vector on a coordinate system is basically irrelevant; all you need is the correct magnitude and correct direction.**



The **resultant** is the sum of two vectors. Use the "tip to tail" method.



We define the difference of two vectors as $v - w = v + (-w)$.

To multiply by a scalar (only magnitude):

- **If $a > 0$, then $a \cdot v$ is a times the magnitude of v.**
- **If $a < 0$, then $a \cdot v$ is a times the magnitude of v whose direction is the opposite of v.**

Work #1 – 3

The symbol $\|v\|$ represents the magnitude of v.

- $\|v\| \geq 0$
- $\|-v\| = \|v\|$
- $\|av\| = |a| \cdot \|v\|$

Remember, the magnitude represents the length of the vector, so it is positive.

Example: If $\|v\| = 2$, what is $\|-4v\|$ equal to? _____

An algebraic vector v is represented as $v = \langle a, b \rangle$ where a and b are real numbers called the components of the vector v.

If $v = \langle a, b \rangle$ is an algebraic vector whose initial point is at the origin, then v is called a position vector, and its terminal point is $P = (a, b)$.

Unit Vectors

Let i have a length of one unit and has a direction corresponding to the direction of the positive x-axis.

Let j have a length of one unit and has a direction corresponding to the direction of the positive y-axis.

Then $i = \langle 1, 0 \rangle$ and $j = \langle 0, 1 \rangle$; $v = \langle a, b \rangle = ai + bj$

Position Vector $v = Q - P$ **(terminal – initial)**

$P = (x_1, y_1)$ and $Q = (x_2, y_2)$, the position vector is $v = (x_2 - x_1)i + (y_2 - y_1)j$

Work #4 – 5

Magnitude

Given $ai + bj$, $\|v\| = \sqrt{a^2 + b^2}$

Work #6 – 7

Add and Subtract Vectors

To add (or subtract) two vectors, add (subtract) corresponding components.

Work #8 – 10

A unit vector has a magnitude or a length of 1.

Suppose v is a vector in standard position and is given by $v = ai + bj$, then a unit vector in the same direction as v is given by $\frac{v}{\|v\|}$.

Work #11

Find a vector given direction and magnitude: $v = \|v\|(\cos a\, i + \sin a\, j)$

Work #12 – 13

If two forces act simultaneously on an object, the vector sum $F_1 + F_2$ is the resultant force.

Work #14