

Calc H Notes Section 2.6 Related Rates

Finding Related Rates

The Chain Rule can be used to find the rates of two or more related variables that are changing with respect to time.

Ex – When water drains from a conical tank, the volume, V , the radius, r , and the height, h , of the water level are all functions of time t .

Start with an equation that relates these variables: $V = \frac{1}{3}\pi r^2 h$

Differentiate both sides with respect to t to obtain the **related-rate** equation.

$$\begin{aligned}\frac{d}{dt}(V) &= \frac{d}{dt}\left(\frac{1}{3}\pi r^2 h\right) \\ \frac{dV}{dt} &= \frac{\pi}{3}\left(r^2 \frac{dh}{dt} + h(2r \frac{dr}{dt})\right) \\ \frac{dV}{dt} &= \frac{\pi}{3}\left(r^2 \frac{dh}{dt} + 2rh \frac{dr}{dt}\right)\end{aligned}$$

From this you can see that the change in volume is related to the rates of change of both r and h .

Ex: In the conical tank suppose the height is changing at a rate of -0.2 foot/minute and the radius is changing at a rate of -0.1 foot/min. What is the rate of change in the volume when the radius is 1 foot and the height is 2 feet? Does the rate of change in the volume depend on the values of r and h ?

$$\frac{dV}{dt} = \frac{\pi}{3}(1^2(-0.2) + 2(1)(2)(-0.1)) = -0.2\pi \text{ ft}^3/\text{min}$$

Guidelines for Solving Related Rate Problems

- 1) Identify all *given* quantities and quantities *to be determined*. Make a sketch and label the quantities.
- 2) Write an equation involving the variables whose rates of change either are given or have to be determined.
- 3) Using the Chain Rule, implicitly differentiate both sides of the equation **with respect to time**.
- 4) After completing step 3, substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.

Example: Ripples in a Pond

A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of disturbed water changing?

- 1) Find the given and what is to be determined:
- 2) Write an equation that involves Area and radius of a circle.
- 3) Differentiate with respect to time
- 4) Substitute values given
- 5) Solve for unknown rate of change

Examples 4-5 p146-147

p149#1-8,13,20-24,26,27

Group presentations: Monday #28,30,32,34,36*,40*,42*,44*,48*

Balloon lab – Wed 1/12; Mathxl due Friday, 1/14 Test Ch 2:5-6 Wed 1/19