Calc H Notes Section 2-3   Derivatives of Products and Quotients

The derivative of a product is <u>not</u> equal to the product of the derivatives. The rule for finding derivatives of products is as follows.

PRODUCT RULE

If $f(x) = u(x) \cdot v(x)$, and if $u'(x)$ and $v'(x)$ both exist, then

$$f'(x) = u(x) \cdot v'(x) + v(x) \cdot u'(x).$$

(The derivative of a product of two functions is the first function times the derivative of the second, plus the second function times the derivative of the first.)

Work p118 examples 1 - 3

Ex 1 Find the derivative of $h(x) = (3x - 2x^2)(5 + 4x)$

Ex 2 Find the derivative of y = $x \sin x$

Ex 3 Find the derivative of y = $2x \cos x - 2 \sin x$

To find the derivative of the quotient of two functions, use the next rule.

QUOTIENT RULE

If $f(x) = u(x)/v(x)$, if all indicated derivatives exist, and if $v(x) \neq 0$, then

$$f'(x) = \frac{v(x) \cdot u'(x) - u(x) \cdot v'(x)}{[v(x)]^2}.$$

(The derivative of a quotient is the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.)

Work Examples 4 - 5

Ex 4 Find y′ if $y = \frac{5x - 2}{x^2 + 1}$

Ex 5 Find y′ if $y = \frac{3 - (1/x)}{x + 5}$.

Rewrite as simple fraction:

Ex Using the Constant Multiple Rule

Original Function Rewrite Differentiate Simplify

a) $y = \frac{x^2 + 3x}{6}$

b) $y = \frac{5x^4}{8}$

c) $y = \frac{-3(3x - 2x^2)}{7x}$

d) $y = \frac{9}{5x^2}$

Derivatives of Trig Functions

$$\frac{d}{dx}[\tan x] = \sec^2 x \qquad \frac{\mathrm{d}}{\mathrm{dx}}[\cot x] = -\csc^2 x$$

$$\frac{d}{dx}[\sec x] = \sec x \tan x \qquad \frac{\mathrm{d}}{\mathrm{dx}}[\csc x] = -\csc x \cot x$$

Examples

Function	Derivative
a) y = x – tan x	
b) y = x sec x	

Much of the work of simplifying the derivative occurs after you differentiate. Simplified derivatives should not have negative exponents and similar terms should be combined.

Higher-Order Derivatives

s(t)	Position function
v(t)=s´(t)	Velocity function
a(t)=v´(t)=s"(t)	Acceleration function

The function a(t) is the second derivative of the position function. This is an example of a higher-order derivative. For example, the third derivative is the derivative of the second derivative. Notation for higher-order derivatives:

First derivative:	y´	f´(x)	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$	$D_x[y]$
Second derivative:	y"	f "(x)	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$	$D_x^{\,2}[y]$

Third derivative: y''' $f'''(x)$ $\frac{d^3y}{dx^3}$ $\frac{d^3}{dx^3}[f(x)]$ $D_x^{\,3}[y]$

Fourth derivative: $y^{(4)}$ $f^{(4)}(x)$ $\frac{d^4y}{dx^4}$ $\frac{d^4}{dx^4}[f(x)]$ $D_x^{\,4}[y]$

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nth derivative: $y^{(n)}$ $f^{(n)}(x)$ $\frac{d^ny}{dx^n}$ $\frac{d^n}{dx^n}[f(x)]$ $D_x^{\,n}[y]$

Example 10 Finding the Acceleration of Gravity

Because the moon has no atmosphere, falling objects on the moon encounter no air resistance. In 1971, astronaut David Scott demonstrated that a feather and a hammer fall at the same rate on the moon. The position function for each of these falling objects is given by

$$s(t) = -0.81t^2 + 2$$

where s(t) is the height in meters and t is time in seconds. What is the ratio of earth's gravitational force to the moon's?

Solution: To find acceleration, differentiate the position function twice.

$s(t) = -0.81t^2 + 2$ position function
$s'(t) = -1.62t$ velocity function
$s''(t) = -1.62$ acceleration function

So the acceleration of gravity on the moon is -1.62 m/sec^2. So the ratio of earth's gravitational force to the moon's is:

$$\frac{-9.8}{-1.62} \approx 6.05$$

Example 11 p124 #3 $h(t) = \sqrt[3]{t}\,(t^2 + 4)$

Example 12 #12 $f(t) = \frac{\cos t}{t^3}$

HW p124 #1-33 eoo,39-53odd,62,65,69,75,77,83-91odd,93-100all,103