Calc H Notes on 2.4 The Chain Rule

The Chain Rule is used when the **base** of an exponent is **NOT** a single variable. In this case the power rule cannot be used.

The Chain Rule is also known as the General Power Rule

$$\frac{d}{dx}[u^n] = nu^{n-1} \cdot u'$$

Ex: $y = \sqrt{x^2+1}$ $u = x^2 + 1$ $y' = \frac{d}{dx}\left[u^{\frac{1}{2}}\right] \cdot u'$ $y' = \frac{1}{2}u^{-\frac{1}{2}} \cdot u'$

Now substitute x^2+1 in for u $y' = \frac{1}{2}(x^2+1)^{-\frac{1}{2}} \cdot (2x)$

Then simplify $y' = \frac{2x}{2(x^2+1)^{\frac{1}{2}}} = \frac{x}{(x^2+1)^{\frac{1}{2}}}$

We have just done the derivative of a composite function using u-substitution.

Another way of looking at it uses the usual composite function notation:

If y = f(g(x)) then y'= f'(g(x))$\cdot$ $g'(x)$

$y = \sqrt{x^2+1}$ $g(x) = x^2 + 1$ $f(x) = x^{\frac{1}{2}}$ $y = f(g(x))$

$$y' = \frac{d}{dx}[(x^2+1)^{1/2}] \cdot \frac{d}{dx}[x^2+1]$$

$$\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) = \frac{x}{(x^2+1)^{\frac{1}{2}}}$$

Ex 2 $f(x) = (3x - 2x^2)^3$ $f'(x) = 3(3x-2x^2)^2(3-4x)$

Ex 3 Find all points on graph of $f(x) = \sqrt[3]{(x^2-1)^2}$ $where\ f'(x) = 0\ or$ $f'(x)\ does\ not\ exist$. That is where the tangent line is horizontal (=0) or tangent line is vertical (undefined – does not exist).

1) Find the derivative of f(x) Rewrite f(x) as $(x^2-1)^{2/3}$

$$f'(x) = \frac{2}{3}(x^2-1)^{-\frac{1}{3}}(2x) = \frac{4x}{3(x^2-1)^{\frac{1}{3}}}$$

2) Horizontal tangent f'(x)=0 if numerator is 0 4x =0 so x=0 plug 0 into original function to find y-coord of point (0,1)
3) Vertical tangent occurs if f'(x) is undefined so the denominator =0

   $3(x^2-1)^{\frac{1}{3}} = 0$, solve for x. Divide both sides by 3, raise both sides to the third power, factor, set factors =0. You should get x = 1, -1. Plug into original function to get y-coordinates. (1,0) and (-1,0).

Ex 4 g(t) =$\frac{-7}{(2t-3)^2}$ g(t)=-7$(2t-3)^{-2}$ g'(t)=14$(2t-3)^{-3}$(2) g'(t)=$\frac{14}{(2t-3)^3}$

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Trig Functions and the Chain Rule

Ex1 y=sin 2x It is often easier to look at these problems using u-sub.

u = 2x, y = sin u, y'=cos u· u' then plug 2x back in for u

y' = cos (2x)(2)

y'=2cos(2x)

Ex2 y = tan3x let u = 3x y = tan u y'=$\sec^2$u·u' Put 3x back in for u

y'=$\sec^2$(3x)·3 y'=3$\sec^2$(3x)

Ex 3 y = cos (x-1) let u=x-1 y=cos u y'=-sin u·u' Put x-1 back in for u

Y'=-sin(x-1)(1) = -sin(x-1)

Ex 4 y = cos ($3x^2$) let u = $3x^2$ y = cos u y'=-sinu· u' Put $3x^2$ back in for u

Y'=-sin($3x^2$)(6x) = -6xsin($3x^2$)

Simplifying Derivatives

Ex 1 f(x) = $x^2\sqrt{1-x^2}$ put in form to differentiate f(x)=$x^2(1-x^2)^{1/2}$ this problem will require a combination of the product rule and the chain rule – proceed with caution!

f'(x)= $2x(1-x^2)^{1/2} + x^2[[\frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)]$ to simplify we are looking for common factors: $x(1-x^2)^{-\frac{1}{2}}[2(1-x^2)^1 + (-1x^2)]$ now simplify within the brackets and move any negative exponents to the denominator.

$$\frac{x[2-2x^2-x^2]}{(1-x^2)^{\frac{1}{2}}} = \frac{x(-3x^2+2)}{(1-x^2)^{\frac{1}{2}}}$$

Repeated Use of the Chain Rule

Ex 1: $f(t) = sin^3 4t$=$(sin4t)^3$ $let\ u = \sin 4t\ , y = u^3$ $y' = 3u^2 \cdot u'$

$u' requires\ the\ chain\ rule.\ f'(t) = 3(sin4t)^2(cos4t)(4)$

Simplified: f'(t)=12($\sin^2$4t)(cos 4t)

Ex 2: y=$sin^2(cosx)$ $y = (\sin(cosx))^2$ $u = \sin(cosx),$ $y = u^2\ , y' = 2u \cdot u'$

$u' requires\ the\ chain\ rule.\ y' = 2\sin(cosx)\ [\cos(cosx)\ (-sinx)]$

= $-2[\sin(cosx)][\cos(\cos x)] \cdot sinx$

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