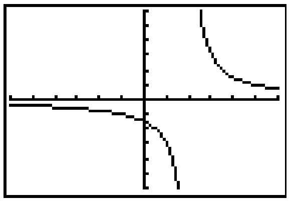
Calc H Notes Section 1.5/3.5 Infinite Limits and Limits at Infinity

A limit in which f(x) increases or decreases without bound as x approaches c is called an infinite limit.

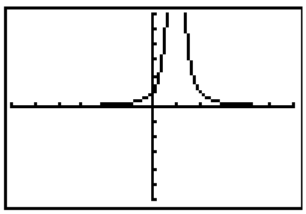
Example: f(x) = $\frac{3}{x-2}$ *(Sketch the graph – it is a transformation of y =* $\frac{1}{x}$*)*



$$\lim_{x\to 2^-}\frac{3}{x-2} = -\infty \quad \lim_{x\to 2^+}\frac{3}{x-2} = \infty$$

When a limit equals $\pm\infty$ *the limit does not exist!*

Example: f(x)=$\frac{1}{(x-1)^2}$



$$\lim_{x\to 1^-}\frac{1}{(x-1)^2} = \infty \quad \lim_{x\to 1^+}\frac{1}{(x-1)^2} = \infty$$

$$\lim_{x\to 1}\frac{1}{(x-1)^2} = \infty$$

Still – the limit does not exist but this notation simply shows that the limit is unbounded as x approaches 1. So instead of DNE, if the limit is unbounded in agreement from both sides, we will use $\pm\infty$.

Definition of vertical asymptote: If f(x) approaches positive or negative infinity as x approaches c from the left OR the right, then the line x=c is a vertical asymptote of the graph of f.

(There will be a vertical asymptote if the denominator =0 when x=c but the numerator does not)

Properties of Infinite Limits

$$\lim_{x\to c} f(x) = \infty \text{ and } \lim_{x\to c} g(x) = L$$

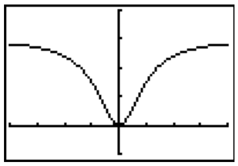
1) Sum/Difference $\lim_{x\to c}[f(x) \pm g(x)] = \infty$

2) Product $\lim_{x\to c}[f(x)g(x)] = \infty, L > 0$ and $\lim_{x\to c}[f(x)g(x)] = -\infty, L < 0$

3) Quotient $\lim_{x \to c} \frac{f(x)}{g(x)} = 0$

Limits at Infinity

This section discusses "end behavior" of a function (as x approaches ∞ *or* $-\infty$).

Ex: f(x)=$\frac{3x^2}{x^2+1}$  *Graphically you can see that f(x) approaches 3 as x increases or decreases without bound.*

Numerically, you could determine the same conclusion by plugging values that get larger in positive direction and negative direction.

X	$-\infty \leftarrow$	*-100*	*-10*	*-1*	*0*	*1*	*10*	*100*	$\rightarrow \infty$
F(x)	*3*$\leftarrow$	*2.9997*	*2.97*	*1.5*	*0*	*1.5*	*2.97*	*2.9997*	$\rightarrow 3$

These limits at infinity are denoted by $\lim_{x \to -\infty} f(x) = 3$ *and* $\lim_{x \to \infty} = 3$

Definition of Horizontal Asymptote

The line y=L is a horizontal asymptote of the graph of f if

$\lim_{x \to \infty} f(x) = L$ *or* $\lim_{x \to -\infty} f(x) = L$

This leads to the conclusion that the graph of any function can have at most two horizontal asymptotes, one to the right and one to the left.

Limits at infinity share many of the properties of limits already discussed in Ch 1: sum/difference, product.

Theorem 3.10 Limits at Infinity

If r is a positive rational number and c is any real number, then

$lim_{x \to \infty} \frac{c}{x^r} = 0.$ *If* x^r *is defined when* $x < 0,$ *then* $\lim_{x \to -\infty} \frac{c}{x^r} = 0$

Examples p193-197 Special note – Example 2 this technique is used quite often to find limits of rational functions.

Example 4 – we will go over in class**

Infinite Limits at Infinity

If f(x) increases or decreases without bound as x approaches ∞ or -∞, then the limit = ∞ or -∞.

Examples 7 & 8 p198